

Hopefully, you are a little comfortable with the relative speed concept now. The concept can come in handy in some circular motion questions too. Today, we will show you how you can use the fundamentals we learned in the last few weeks to solve questions involving moving in a circle. Let's try a couple of GMAT questions to put what we learned to use:

Question 1: Two cars run in opposite directions on a circular path. Car A travels at a rate of 4π miles per hour and car B runs at a rate of 6π miles per hour. If the path has a radius of 8 miles and both the cars start from point S at the same time, how long, in hours, after the cars depart will they meet again for the first time after leaving?

- (A) 1 hr
- (B) 1.2 hrs
- (C) 1.5 hrs
- (D) 1.6 hrs
- (E) 1.8 hrs

Solution:

When two objects travel in opposite directions, their relative speed is the sum of their speeds. So the relative speed of the two cars is $(4\pi + 6\pi)$ mph = 10π mph

The length of the path is given by the circumference of the circle i.e. $2\pi \cdot 8 = 16\pi$

The two cars leave together from S. When they meet again, together they have covered the length of the path i.e. 16π miles.

Time taken to cover a full circle by both together = Distance Covered/Relative Speed = $16\pi/10\pi = 1.6$ hrs

The concept is exactly the same. The only thing different is that you need to calculate the length of the path here. Let's look at a trickier question now.

Question 2: Bob starts at point X and runs clockwise around a circular track at a constant rate of 2 mph. Ten hours later, Alan leaves from point X and travels counter-clockwise around the same circular track at a constant rate of 3 mph. If the radius of the track is 10 miles, for how many hours did Bob run from the time he started to the time he passed Alan for the first time and put another 12 miles between them (measured around the curve of the track)?

- (A) $4\pi - 1.6$
- (B) $4\pi + 8.4$
- (C) $4\pi + 10.4$
- (D) $2\pi - 1.6$
- (E) $2\pi - 0.8$

Solution: The question stem is rather long. You should process one statement at a time. Let me break the question stem into multiple statements. Let's learn how to handle a question one step at a time.

“Bob starts at point X and runs clockwise around a circular track at a constant rate of 2 mph.”

- Ok, so Bob covers 2 miles every hour moving clockwise. We don't know the track length yet.

“Ten hours later, Alan leaves from point X and travels counter-clockwise around the same circular track at a constant rate of 3 mph.”

- Alan is faster and covers 3 miles every hour counter clockwise. Since their directions are opposite, their relative speed = $2+3 = 5$ mph. We still don't know the track length so we cannot say where Bob was when Alan started. All we know is that in 10 hrs, Bob traveled 20 miles.

“If the radius of the track is 10 miles,”

- Now we know the track length. It is $2\pi r = 2\pi \times 10 = 20\pi$. It is greater than the 20 miles that Bob covered in 10 hrs so Bob has not finished one round. Of the 20π , he has covered only 20 miles.

“for how many hours did Bob run from the time he started to the time he passed Alan for the first time and put another 12 miles between them (measured around the curve of the track)?”

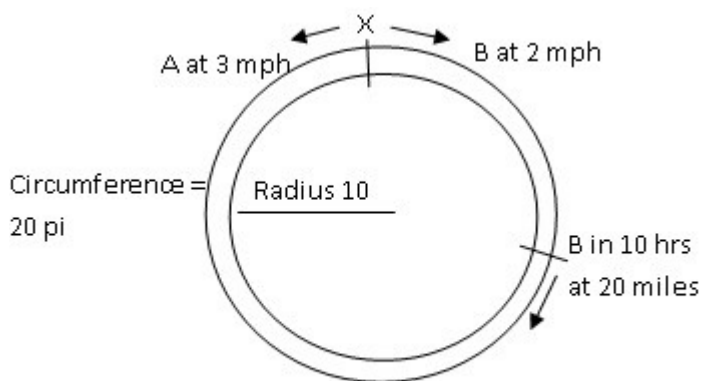
- To pass each other for the first time, Alan and Bob together need to cover the remaining distance on the circle i.e. $20\pi - 20$. To create a distance of another 12 miles, they together need to travel 12 miles more away from each other.

Time taken = Total distance to be traveled/ Relative Speed

Time taken = $(20\pi - 20 + 12)/5 = 4\pi - 1.6$ hrs

Bob must have been traveling for $4\pi - 1.6 + 10 = (4\pi + 8.4)$ hrs

Another way to approach this question is using a diagram:



Radius of the track is 10 miles so circumference is 20π i.e. the total length of the track.

Bob starts from X and travels for 10 hrs clockwise at 2 mph i.e. he travels 20 miles. Now Alan starts from X counter clockwise. Distance between Alan and Bob is $20\pi - 20$.

Now, to meet, they have to together cover this distance plus 12 miles more which they have to put between them.

Time taken to cover this distance by them = $(20\pi - 20 + 12)/(3 + 2) = 4\pi - 1.6$ hrs

Bob has been traveling for $10 + 4\pi - 1.6 = 4\pi + 8.4$ hrs

I hope you see that the questions become very manageable if you use relative speed.